PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Shear Patterns in an Unstable Layer of Air

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Phil. Trans. R. Soc. Lond. A 1934 **232**, 285-296
doi: 10.1098/rsta.1934.0008

VIII. *Shear Patterns in an Unstable Layer of Air.**

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(*Communicated by* Sir Gilbert Walker, *F.R.S.*)

(Received April 12, 1933—Read November 2, 1933.)

[Plates 2–5.]

Introduction.

Work on this subject has recently been done by Walker and Phillips.† In their experiments air was drawn by an electric fan through a channel bounded by a hot sheet of metal beneath, separated by about 6 mm. from a cold sheet of plate glass above. They produced polygons in air at rest, and longitudinal and transverse vortices when the fan was working; but as they pointed out there was a double shear in their apparatus which rendered their conditions not identical with those in the sky.

Experiments with a single shear in liquids have been performed by Idrac, Mal, and others; longitudinal strips have been obtained and a transitional pattern, but no transverse rolls.

The present work was undertaken at the suggestion of Sir G. T. Walker to investigate the effects in a thin layer of air subjected to single shear; to see whether, as predicted by him, the longitudinal and transverse rolls appear in this case also, and if the adjacent transverse rolls rotate in opposite directions. In addition it was hoped to throw more light on the formation of other patterns observed in the sky, notably polygons with ascent as well as those with descent in the centres, and rectangular patterns. The latter were expected to form an intermediary between the longitudinal and the transverse rolls.

During the course of the present experiments the previous experiments with air subjected to double shear were repeated, and the results confirmed.

Part I.

1. In order to obtain single shear conditions it was necessary for one of the bounding surfaces of the layer to move. An apparatus was constructed with a channel depth of 1 cm., the upper surface being an endless rubber belt running on rollers, and the lower

* Being the work performed for the degree of M.Sc., in the University of London.

† 'Quart. J. R. Met. Soc.,' vol. 58, p. 23 (1932).

 [Published November 13, 1933.

surface a sheet of glass warmed by electric heaters. With this arrangement longitudinal vortices were produced with the band in motion, but with the band at rest irregular rolls were formed, fig. 13, Plate 3, instead of polygons. WALKER and PHILLIPS have shown that polygons are always formed in the absence of shear, and the failure to obtain these with the above arrangement led to some experiments described in Part II. It was found that an extreme uniformity of depth of the channel was necessary, and this was prevented by sag in the moving band.

It was accordingly decided to use a strip of plate glass of size 8 feet by 8 inches as the upper surface, and to draw it over the channel to produce a shear. The apparatus now took the following form, fig. 1 : the lower surface of the layer was an iron plate 36 inches

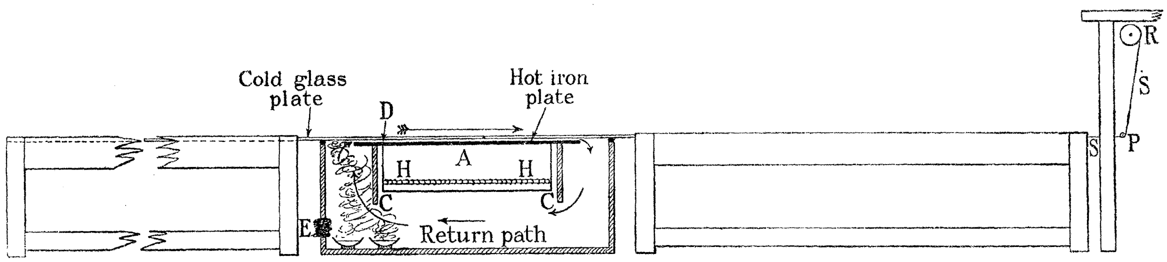


FIG. 1.

by 8 inches by $\frac{1}{4}$ inch thick, painted black to form a contrasting background for white smoke, and warmed by electric heaters. At each end of the plate were placed two trestles 6 feet long, fitted with guides to take the glass strip. The trestles and iron plate were adjusted so that the strip could slide from one trestle to the other over the iron plate, leaving a parallel space 8 mm. deep between them. Strips of felt formed the long sides of the channel while at first the ends were left open. A string S, fastened to the end of the glass strip, passed under a pulley P at the same height as the strip, and was wound on to a roller R. The string was attached to an improvised wooden clamp which held the glass plate between its jaws, small pieces of rubber being placed between the wood and the glass. The roller R was driven by a 1/8th H.P. shunt-wound motor through a 50 : 1 reduction gear and pulleys giving 150 : 1 reduction in all.

The method of working was to start with the glass plate just covering the channel, smoke being then introduced the plate was immediately drawn across ; when the plate had reached the limit of its travel the belt was removed from the pulleys to enable the glass plate to be returned to its former position.

It was found that the flow was disturbed by draughts in the room making it necessary that the layer should be completely closed. The ends could not be closed directly as this would cause a complicated shear, arrangements were therefore made to allow the displaced air to return to the beginning of the channel via another channel placed on a level with and alongside the first. With this arrangement longitudinal rolls and chains of polygons, fig. 8, Plate 2, could be formed, but not transverse rolls ; the difficulty led to a repetition of the experiments with double shear. It was found that transverse vortices must be formed *ab initio*, no other pattern would change into them. More care was then taken to arrange a straight flow into the end of the channel in order that there should be no opportunity for an irregular pattern to form first.

Accordingly a new heater box was constructed, fig. 1. The centre portion A, 30 inches by 7 inches by 6 inches deep, made of zinc nailed inside a wooden framework, contained four long heater elements H, and on it rested the iron plate overlapping by 3 inches at each end; this was done so that the end of the plate should be cool, enabling the air stream to become more regular before instability was set up at D. The return path was arranged underneath as shown, while at CC were placed pieces of heat insulating board in an attempt to minimize the circulation of air in the chambers at each end. At E a hole was left for the introduction of an evaporating basin containing ammonia, and another containing hydrochloric acid; the hole could be closed with a wad of cotton wool.

On using this apparatus it was again found that polygon chains and longitudinal rolls were all that could be obtained.

As polygons were formed at the slowest speed of the plate it was supposed that the plate would move slowly enough to form transverse rolls. However, on analysing the motion of a system of polygons (see Part I, section 4) it was noticed that polygons arranged as in (*a*), fig. 2, can be drawn into longitudinal rolls, while those arranged as

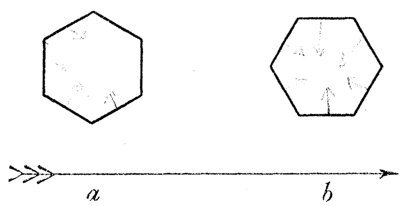


FIG. 2.

in (*b*) cannot. This being so it was not unreasonable to suppose that different values of the shear would orientate the polygons differently; also that polygons (*a*) being associated with longitudinal rolls might even appear at a faster speed than do the transverse rolls: when the speed of the plate was reduced to 0·5 cm./sec. the transverse rolls were obtained; they are shown in fig. 3, Plate 2.

In all the shear photographs Plates 2 and 3, the motion of the upper surface of the layer was from left to right.

2. There was a photographic difficulty in obtaining an adequately exposed plate; on account of the movement the longest exposure that could be given was 1/5th sec. Undoubtedly the best illumination is provided by a beam of light thrown transversely through the layer, but the necessary window in the side of 8 mm. depth was not easy to arrange. A beam of light from an arc lamp was thrown on to the smoke at an angle of about 30° to the horizontal; this arrangement, however, decreased the contrast by illuminating the lower boundary.

The photographs shown were obtained with ammonium chloride smoke, using soft gradation panchromatic plates with an exposure of 1/5th sec. at F/4·5.

There was also another difficulty due to general air currents. When the glass strip had moved a foot or so the hot air carried along by it accumulated in the chamber at the end of exit, and tended to come back along the channel in the reverse direction. This difficulty was overcome by blowing through the return path, before the glass strip was returned between each experiment, to remove the hot air ; the flow was undisturbed for a few feet of the travel of the plate during the subsequent experiment, as demonstrated by the pattern moving with half the speed of the plate.

3. The experimental photographs shown represent the appearances obtained at approximately the actual size.

Using a thickness of 8 mm. for the air layer the following results were obtained : at a plate speed of 0·5 cm./sec. transverse rolls appeared as in fig. 3. Adjacent rolls rotated in opposite directions, as with double shear. These were apparently all in the same plane, but owing to the small amount of smoke in the rolls it was difficult to be sure on this point.

At a slightly greater speed these rolls thickened periodically along their length as in fig. 4, Plate 2.

At a speed of 1 cm./sec. a square pattern was occasionally seen, oriented with diagonals along and across the line of shear, though only a small portion of the channel showed the pattern at any one time as shown at the top of fig. 5, Plate 2. This pattern was finally obtained in the following manner :

With the plate at rest tobacco smoke was blown into the layer very gently. Irregular polygons formed immediately and in course of time became more regular. When several attempts had been made and a fairly regular set was obtained arranged as in fig. 2 (*b*), the plate was set in motion at 1 cm./sec. The polygons immediately changed through the transitional patterns of fig. 6, Plate 3, to the square pattern shown in fig. 7, Plate 3. Starting from this same arrangement of polygons and setting the plate moving at 0·5 cm./sec. rounded cells of fig. 6, were produced and not the transverse rolls. The formation of the favourable system of polygons depended on chance, but from this system the rectangular and transitional patterns could be obtained every time.

At a speed of between 1·5 cm./sec. and 2 cm./sec. fairly regular polygons moved down the channel as shown in fig. 8.

At a speed greater than about 2 cm./sec. the longitudinal rolls of fig. 9, Plate 2, were formed. In this as in the double shear experiments adjacent rolls rotated in opposite directions.

The speeds given above are necessarily only approximate, because complete regularity of conditions could not be achieved, in fact, many of the photographs show two of the patterns at the same time ; these photographs are useful in helping to piece together the complete transformation from polygons to longitudinal rolls. This transformation was not achieved experimentally, and was partly due to the fact that the smoke settled or dispersed too quickly, and to the fact that conditions were not sufficiently regular.

An attempt was made to get the patterns with different depths of channel. At a thickness 1 mm. greater than the successful 8 mm., *i.e.*, at 9 mm. transverse rolls could be obtained on occasions, but these were extremely irregular as fig. 10, Plate 2, taken at this thickness will show. The plate speed in this case was slightly greater than that for transverse rolls with 8 mm. thickness.

With a thickness of 7 mm. the air layer could not be made unstable as the necessary temperature difference was greater than the apparatus would give. Thus it was not possible using the apparatus as it stood, to find the relationship between the shear producing a given pattern and the depth of the layer.

During the experiments it seemed that the patterns, and the shear to produce a given pattern were apparently unaffected by the value of the temperature difference between the top and bottom of the layer, so long as there was complete instability, but further information is needed on this point.

4. Before discussing the results further, attention must be drawn to the following point. It has been shown experimentally by other workers that polygons caused by instability, whether in liquids or in gases, tend to be hexagonal in shape when conditions are uniform.

Consider the system of hexagons shown in fig. 11. The arrows represent the direction of flow of the fluid at the upper surface for a gas. To turn this system into rolls parallel

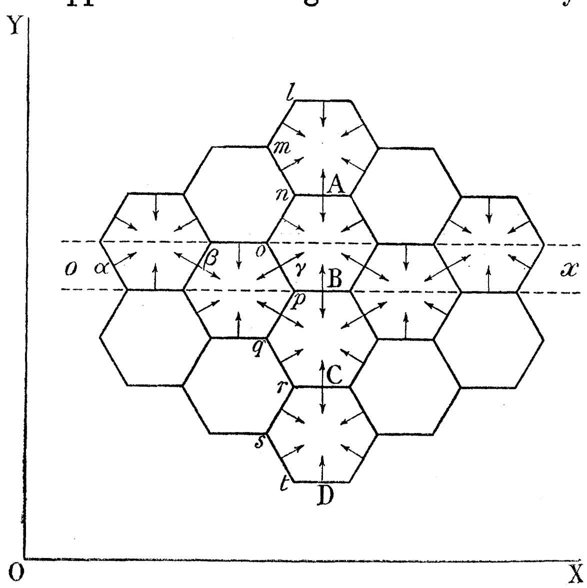


FIG. 11.

to OY, with adjacent rolls rotating in opposite directions, a straightening of the lines corresponding to *l*, *m*, *n*, *o*, *p*, *q*, etc., is necessary with suppression of the flow at A, B, C, etc. To turn this system into a set of similar rolls parallel with OX, a complete re-arrangement is necessary. For if we consider the section *ox*, adjacent portions of this are rotating in opposite directions so that no roll is possible, *i.e.*, looking in the direction *ox* the section αβ is rotating anticlockwise, the section βγ clockwise. Any other section parallel with this is more complicated still.

Thus we can see that the patterns derivable from a system of polygons depend on the orientation of the system.

Bearing this in mind it is possible to piece together the complete transformation from polygons to longitudinal rolls, from a consideration of the figs. 3 to 9, Plates 2 and 3. This is represented diagrammatically in fig. 12, which must be compared with the photographs.

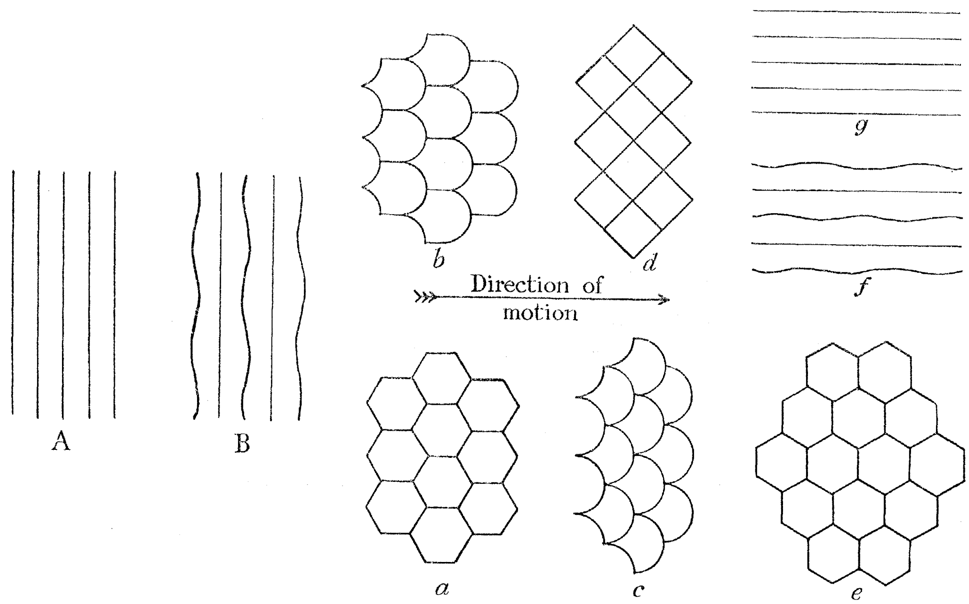


FIG. 12.

Starting at rest with polygons orientated as shown in fig. 12 *a*, and considering the shear increasing, the next stage is 12 *b*, followed by 12 *c*, at a shear corresponding to a plate speed of between 0·5 and 1 cm./sec. Patterns 12 *b* and 12 *c* are seen together in fig. 6. No. 12 *c* is placed after 12 *b*, *i.e.*, at a greater shear, since 12 *c* and 12 *d* appear together in fig. 7, taken at 1 cm./sec. On the right-hand side of fig. 7, can be seen the type *e*—the polygon chains. This pattern appears also in fig. 8, taken at about 1·5 cm./sec., together with types *f* and *g*. The pattern *g* is shown in fig. 9, taken at a greater speed. Thus the order of appearance of the patterns *e*, *f* and *g* is determined.

It is seen that the transverse rolls are not part of the scheme; the fact that they were generated independently and could not be transformed from anything else confirms this; there is also the fact that at a speed of 0·5 cm./sec., both the transverse rolls and the pattern *b* were obtainable according to whether the pattern started *ab initio* or whether it started from *a*. The transverse rolls appear, therefore, to occupy a unique position in the series of patterns. The fact that the transverse rolls have not yet been obtained in a liquid lends further support to the statement.

Attention is drawn to the obvious pairing of the rolls. It appears that whatever the form of the pattern, the air layer is divided into cells, completely closed, with descent in the centre of each.

There is one final point to be noticed: of all the patterns the only ones which are not symmetrical about a line perpendicular to the shear are *b* and *c*. The other patterns all define direction, but not sense; patterns *b* and *c* define sense in addition. These patterns distinguish between the relative motion of the lower and upper surfaces.

In air the convex sides of the curved edges face downstream, while one of MALS'* photographs shows that for a liquid they face upstream.

PART II.

It was mentioned in Part I, § 1, that the difficulty of obtaining polygons at that stage of the experiments led to a subsidiary investigation to determine the conditions under which they form; these investigations are now described.

5. The first experiments were made with a parallel layer of air about 6 mm. thick, bounded below by a piece of sheet tin, and above by a sheet of glass. The top and bottom were kept apart by wooden spacers, the whole forming a flat box about 20 cm. square and 6 mm. deep. Except for a glass tube for the introduction of tobacco smoke the box was rendered airtight with plasticine; the blackened tin bottom was heated by an electric heater.

No system entirely composed of polygons was seen at any time, but instead there were irregular rolls with an occasional polygon here and there; after a while the polygons disappeared, the appearance now being that shown in fig. 13, Plate 3.

With any thickness greater than about 1 cm. the air became unstable with a very small temperature difference between the top and bottom surfaces. No definite patterns were visible at all in these experiments, but the smoke could be seen to be in violent motion.

A layer about 3 mm. thick was set up, and fig. 14, Plate 4, shows the appearance of the polygons which were formed. The air motions were extremely slow. The pattern is apparently revealed on account of the settling of the smoke: a layer of clear air is left at the top and is carried down through the smoke layer by the descending currents; this affords a very sensitive method of indicating the motion.

In fig. 14 each smoke patch is surrounded by clear air; this suggests that the polygons have ascending centres, but a closer examination shows a clear dot in the centre of most of them suggesting descending centres; further information is needed on this point.

With this thickness also an interesting photograph of shear forms was taken, fig. 15, Plate 4. These forms were produced when the smoke, gently blown in at one corner, was spreading out across the trough, producing a shear. Transverse, rectangular and

* Beitr: Phys. frei. Atmos., vol. 17, fig. 14, p. 45 (1930).

longitudinal patterns are clearly shown, the rectangular pattern in this case is oriented along and across the line of shear and is thus different from the square pattern of the single shear experiments; there are indications, however, of the diagonally arranged rectangular pattern at the top of the figure.

There is a further point to notice in connection with this experiment; the maximum velocity occurs between two vortices, and here, if the motion is upwards, the smoke may be lifted to the glass surface above. This actually takes place along a line down the centre of each smoke patch; in the photograph the line shows lighter than the rest of the patch. At the left-hand bottom corner of fig. 15 these lighter lines are arranged in the form of a polygonal network, though this bears little relationship to the clear spaces on which it is superimposed. It shows conclusively that in this case the cells have ascending edges, though the motion may be more complicated than that in a simple cell.

With the exception of the transitional patterns all the types appear on the one photograph.

With a layer of greater depth than about 3 mm. polygons could not at first be formed. It was thought that the emissivity of the bounding surfaces might have some effect; accordingly a layer of depth 8 mm. was set up using a piece of stainless steel mirror 6 inches square as the lower surface and carefully planed strips of wood as spacers. With this arrangement polygons having descent in the centres were obtained when the bottom was sufficiently hot, figs. 17 and 18, Plate 4. However, when the mirror was blackened with soot polygons were still formed; the results were the same with the apparatus inverted so that the mirror was above, in addition, when using these spacers polygons were formed between glass surfaces; thus the nature of the bounding surfaces made no difference qualitatively to the result. A comparison of figs. 17 and 18 with those from the shear experiments shows that the patterns in each are very much alike; in figs. 17 and 18 can be seen the patterns *a*, *b*, *c* and *d* of fig. 12. Thus it appears that the rolls so often obtained without deliberate shear are, nevertheless, due to shear caused by general thermal circulations; the thinner the layer the greater is the effect of viscosity in damping out the shears set up by the irregularities of thickness and heating, and hence the greater the ease of obtaining regular patterns. The particular set of wooden spacers were decidedly regular in thickness and were thin enough to make the damping influence of viscosity effective.

6. In the previous section the conditions necessary for the formation of polygons were discussed; it now remains to describe the various stages anterior to the formation of these polygons.

Let us consider an air layer 8 mm. in thickness and heated from below; when the heater is switched on and tobacco smoke is blown in there appear, momentarily, a number of columns of clear air which can be seen to be ascending from the lower surface, fig. 19, Plate 4; this occurs only when the bottom is slightly warm. The instability disappears almost immediately and the smoke forms a uniform layer in the space. After this:

(*a*) The smoke comes away from the edges of the layer leaving a clear space about 1 cm. wide all round. This indicates an ascent in the centre of the layer with descent at the edges. The clear space is due to the fact that the smoke settles; the velocity of the air is very slow.

(*b*) Soon after the separation of the smoke from the edges a vortex roll forms along each edge in the clear space.

(*c*) The smoke collects at several places and thins out in others.

(*d*) The number of these regions increases. This shows that as the bottom gets hotter the number of centres of ascent gets greater.

(*e*) At several places round cells appear, more or less regularly disposed; they have very slow ascent in the centres, and are shown in fig. 16, Plate 4.

(*f*) The number of these rings increases until the whole layer becomes completely unstable and is filled with a mixture of polygons and rolls.

(*g*) Finally the rolls turn into polygons which have descent in the centres, leaving the space completely filled with polygons, see figs. 17 and 18.

If the conditions are not regular the initial round rings of fig. 16 do not appear, but instead the pattern of fig. 20, Plate 5, is shown.

It has not been possible to find out exactly how the change over from ascent to descent takes place because it is difficult to watch the direction of circulation and the form of the pattern at the same instant. It is suggested that the change takes place via the roll form, the rolls first pairing about ascending columns and then changing to pairs about descending columns.

There is one other experiment to describe, the purpose of which appears in the theoretical Part III. Tobacco smoke was blown into the layer, and sufficient time was given to allow it to settle and leave a layer of clear air at the top. The upper glass surface was then suddenly cooled by rubbing it all over with a piece of ice, and almost immediately two clearer spaces formed towards the centre of the layer with concentration of smoke between them and round the outer edges. Thus the motion was a descent in columns.

Part III.

An attempt should now be made to give a qualitative explanation of the contrast observed between the directions of vortex motion in a gas and in a liquid.

7. In the experiments described in Part II it was found that:

(*a*) When the under surface of an air layer is suddenly heated, thus creating a thin layer of less density at the lower surface, the instability takes the form of a number of ascending columns.

(*b*) When the upper surface of an air layer is suddenly cooled the instability takes the form of a number of descending columns ; in this case the instability is definitely at the upper surface.

If, then we distinguish between the initial temporary motions set up by heating or cooling and the steady state which ensues, it appears from experiments (*a*) and (*b*) that during the initial period the motion of the gas is distinctly in columns away from the unstable surface.

(*c*) When conditions have become steady, polygons with descent in the centres are formed. Now owing to the mixing effect of the circulations the main body of the gas must be more or less at a uniform temperature, but the temperatures of the layers of gas against the top and bottom boundaries must approximate to the temperatures of those boundaries. Since the motion is a descent in columns from the upper surface it is evident that it is the upper surface layer which is unstable, or at least is more unstable than the lower layer.

It is well known, however, that with a liquid when a steady state is reached polygons with ascent in the centres are formed, that is, with a liquid it is the lower surface layer which is the more unstable.

Accordingly the question at issue is why, with a gas, is the cold surface more unstable than the hot surface, while for a liquid it is the reverse.

It is clear that an increase of viscosity makes the layer more stable. Now the viscosity of a gas increases with rise of temperature, whereas, for a liquid it is the reverse. Thus for a gas the upper, and for a liquid the lower, boundary layer is the more unstable. Considering the magnitude of the viscosity changes the viscosity of air is approximately proportional to the absolute temperature to the power 0·75, whilst for liquids the variation is, in general, very much larger than this, so the variations of viscosity fit with the suggested explanations.

In order to test this theory we may apply it to obtain a descent in columns from the upper surface of a liquid.

Hot water was poured into a cigarette tin to the depth of about 1 cm. ; when the disturbances had died away a little cold milk was carefully poured into the water. The cold milk sank to the bottom forming a white layer below and leaving clear water above. If now the free surface of the water was cooled by blowing upon it, a number of round holes appeared in the milk layer, fig. 21, Plate 5, clearly demonstrating the descending columns.

The increase in the number of centres affords a reason for the formation of hexagons. With a perfectly uniform layer, from consideration of symmetry, the cells will at first be round, of equal diameters and uniformly distributed. As the number of centres increases a state will be reached when the cells touch, each ring then having six equal rings symmetrically disposed around it. Any closer packing can now only take place by the elimination of the vacant spaces between the rings ; there will be a further squeezing together and a uniform hexagonal pattern will be formed.

Part IV.

Application of the Results to Cloud Forms.

8. The longitudinal, transverse and rectangular patterns may all be seen in the sky in cloud formations. Photographs of these clouds have been published by Walker and others. In these publications there has been no reference made to the fact that the square pattern is at times placed diagonally to the shear. I have seen the pattern in the sky associated closely with transverse rolls and disposed diagonally to them, but unfortunately a photograph taken of the cloud was not very satisfactory. The square pattern is often seen in a lenticular cloud bank, as shown by Clarke,* arranged with the diagonals along the long axis of the bank. In view of the fact that in cloud banks of this type ripples are often seen which are clearly transverse to the shear and are parallel to one set of diagonals, there is little doubt that the square pattern is often aligned this way in the sky.

The "transitional" pattern of fig. 6 also occurs in the sky; examples of this are shown by Clarke, Plate 18 *b*, and in fig. 22, Plate 5, in this paper; this cloud form has been seen to develop into the rectangular pattern.

In the laboratory there is a greater tendency to form cells having descent in the centres in a layer of air while in the sky both ascent and descent are found. It is highly improbable that viscosity, which largely controls the effects in the laboratory will play much part in controlling the instability of layers in the sky; thus considerable instability at the upper or lower surfaces of an atmospheric layer are both possible.

Fig. 23, Plate 5, shows transverse clouds apparently of reticular† type and that pairing of rolls also occurs in the sky. Fig. 24 (*a*) shows an end view of rolls of normal type and fig. 24 (*b*) of reticular type. The shaded parts represent cloud, which, of course, forms in the upward currents. The difference between the two types is apparent; if the humidity is great, however, the clouds A and B may join and appear as one. Thus the reticular rolls only should appear double in the sky.

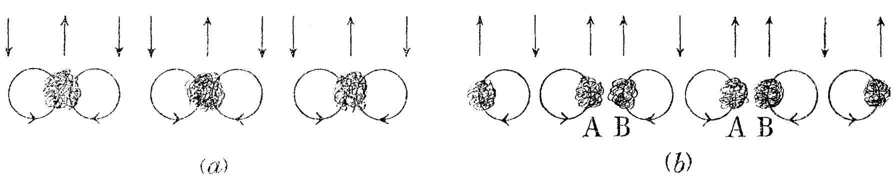


Fig. 24.

The experiments described in Part III suggest that unstable clouds of the normal type, *i.e.*, clouds due to ascent in the centre of a cell, are formed by instability produced by

* "Clouds," Plate 21, *b*, London, 1920.

† For meaning of the term "reticular" see 'Quart. J. R. Met. Soc.,' vol. 52, p. 27 (1932).

the layer being warmed from beneath, while clouds of the reticular type, having descent in the centre of a cell, are formed when the layer is cooled from above.

I take this opportunity to express my sincere thanks to Sir G. T. WALKER for suggesting the work and for his very valuable help and advice during its progress. My thanks are also due to Professor F. T. HILL for his advice in connection with building the apparatus.

SUMMARY.

It has been shown that with an unstable air layer subjected to a single shear:

(*a*) Longitudinal and transverse rolls may be obtained as with a double shear.

(*b*) A square pattern may be obtained with its lines inclined at 45° to the shear.

(*c*) Between the polygonal and square patterns there is a transitional pattern.

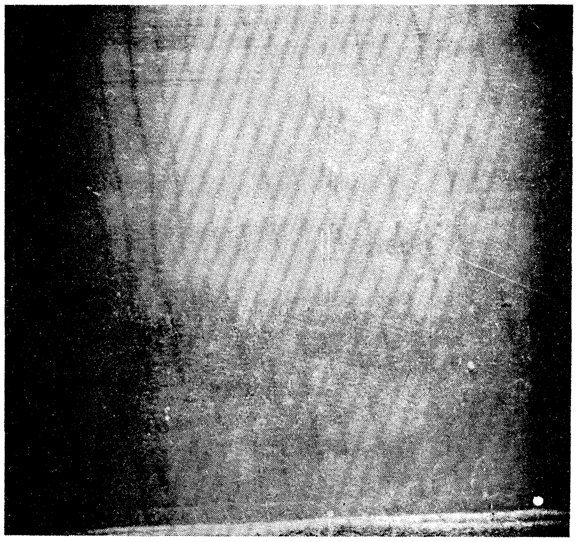
(*d*) With a certain orientation the polygonal pattern is stable under the influence of a single shear having a suitable value.

In a layer of air not subjected to shear:

(*e*) Instability in the form of polygons does not occur suddenly, but the number of centres of motion increases steadily; the nature of the bounding surfaces makes no difference qualitatively, to the effects.

(*f*) Ascending columns may be produced in air by heating the lower surface and descending columns in water by cooling the upper surface; an explanation of the facts is put forward.

The meteorological effect of the work is to give experimental evidence that the longitudinal and transverse rolls of a cloud sheet can be produced by a single shear, and has brought to light a new pattern in the form of a diagonal arrangement of squares. The experiments also suggest that normal and reticular cloud types are produced by heating from below and cooling from above respectively.



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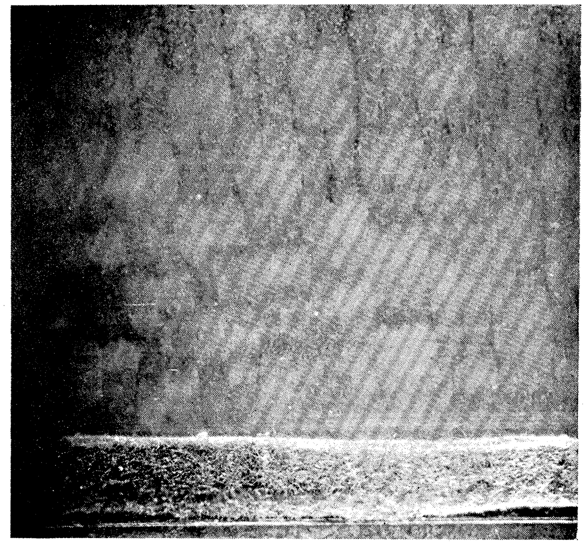

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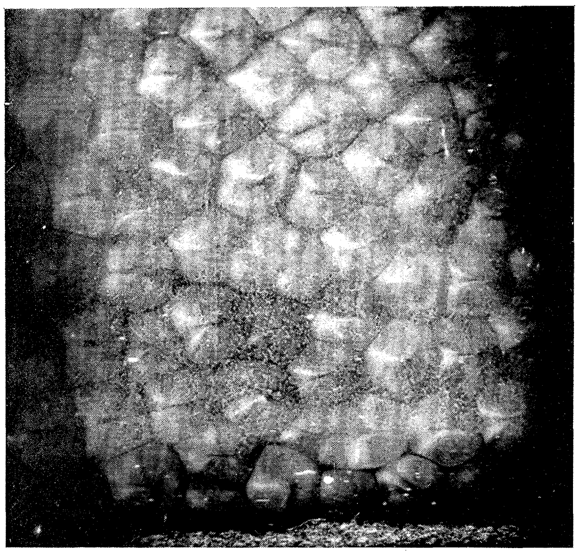

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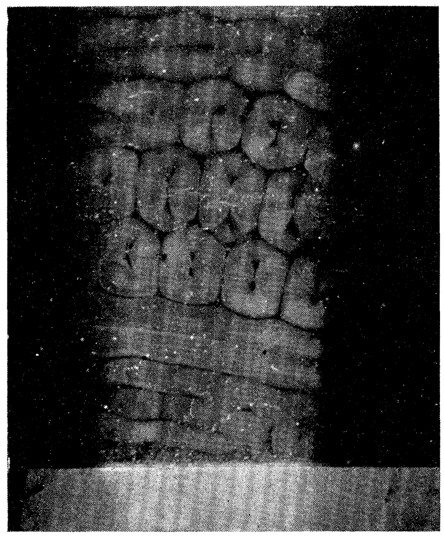

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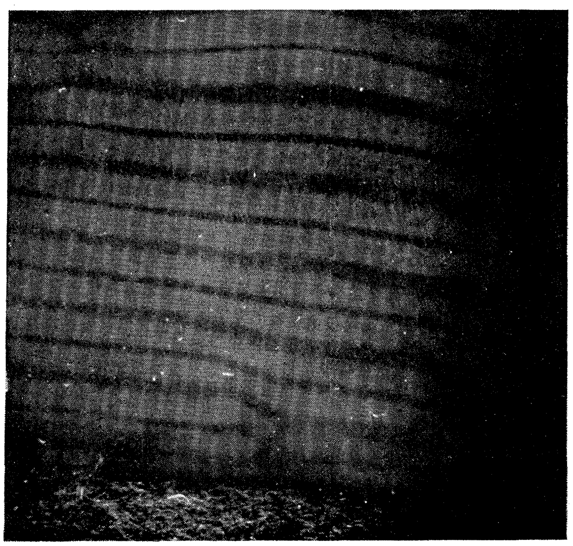

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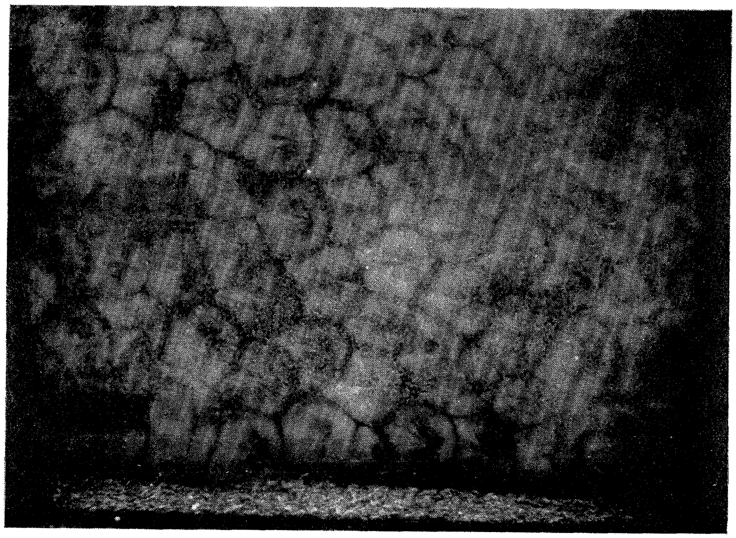

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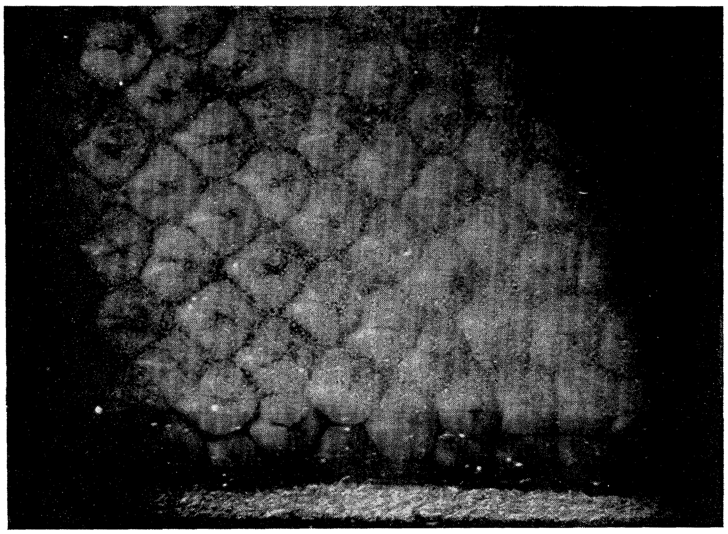

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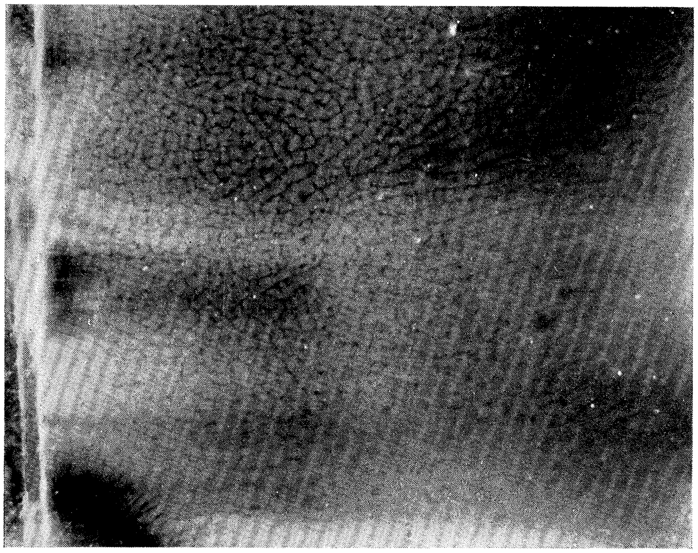

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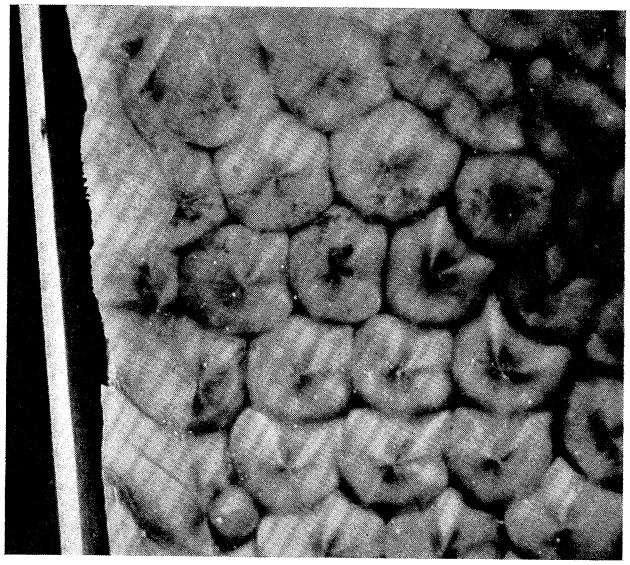

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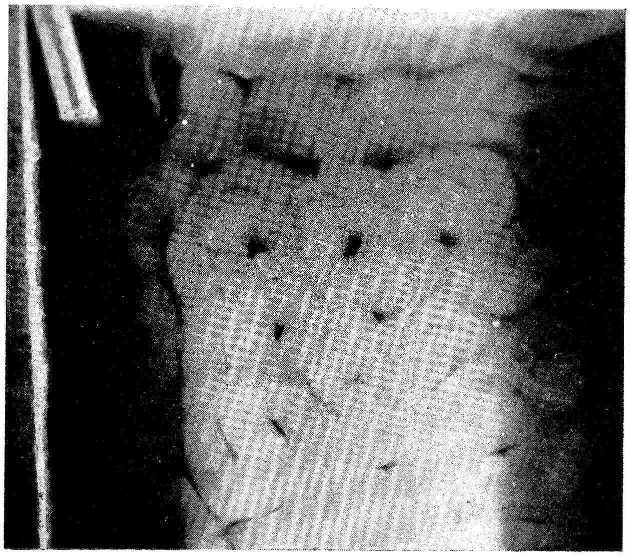

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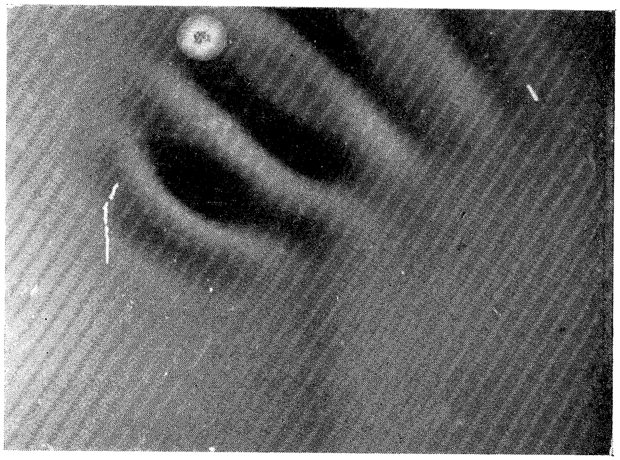
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES



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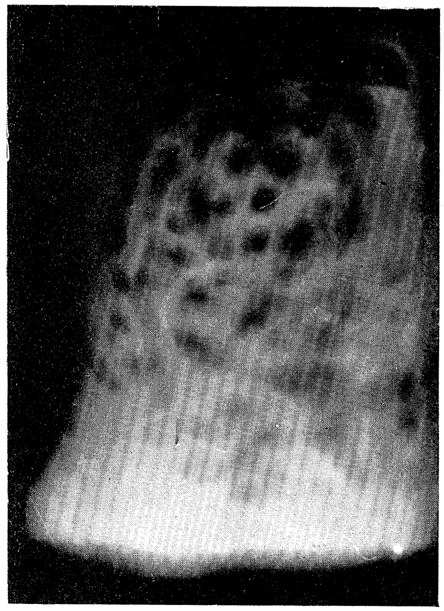

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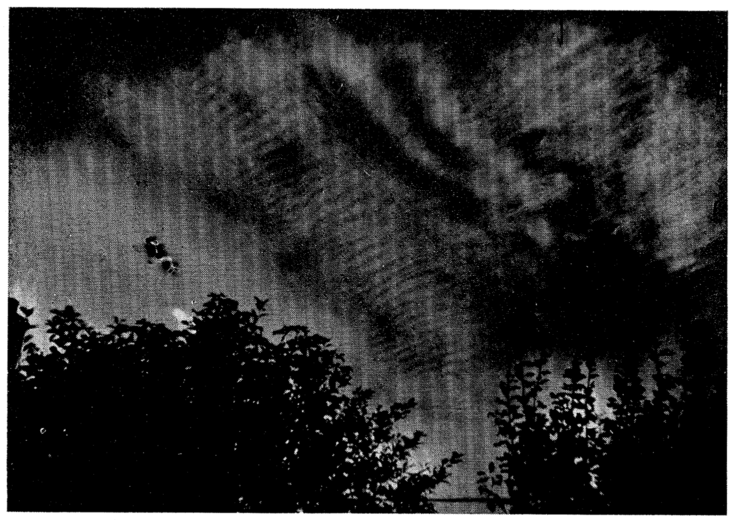

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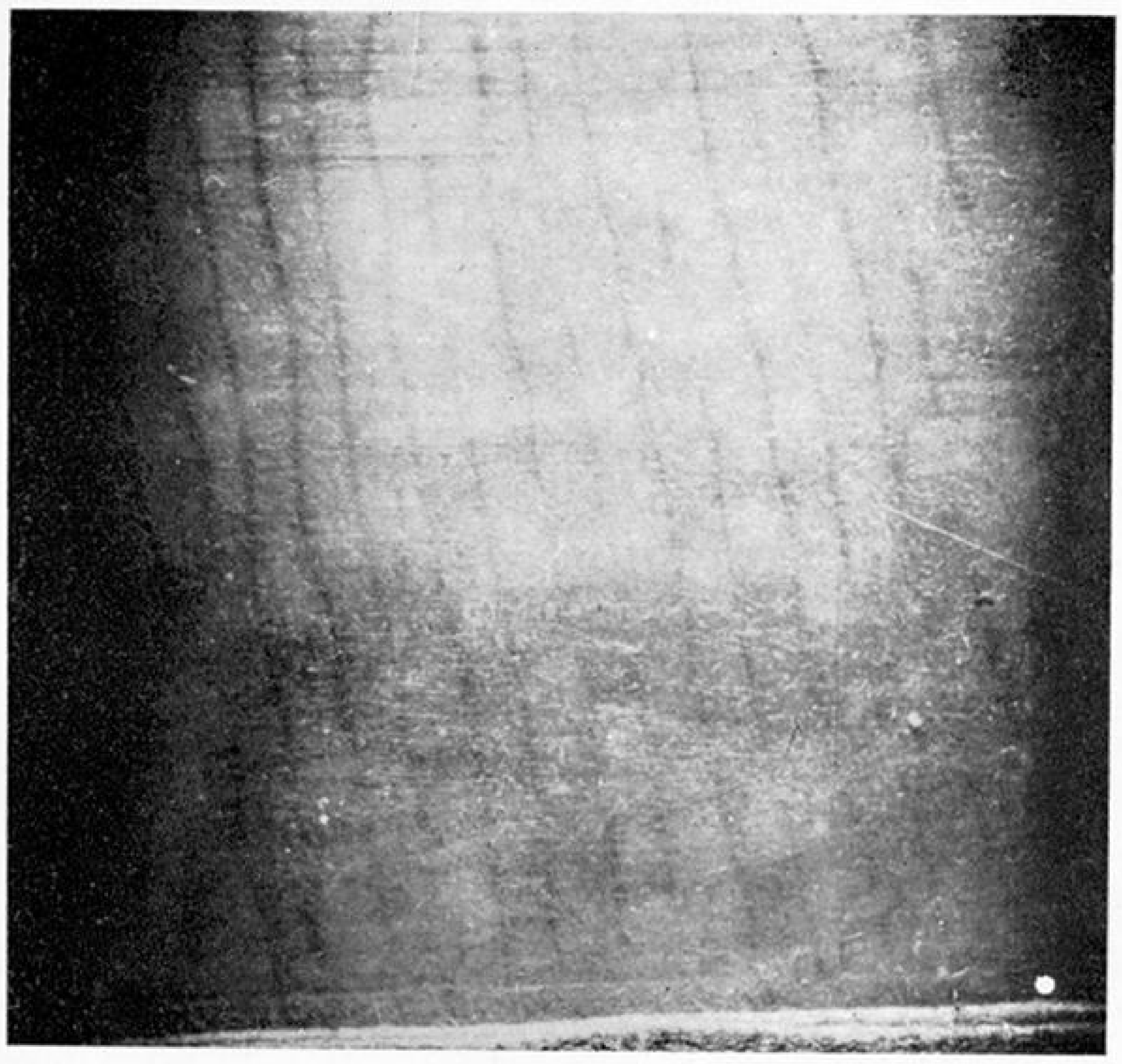

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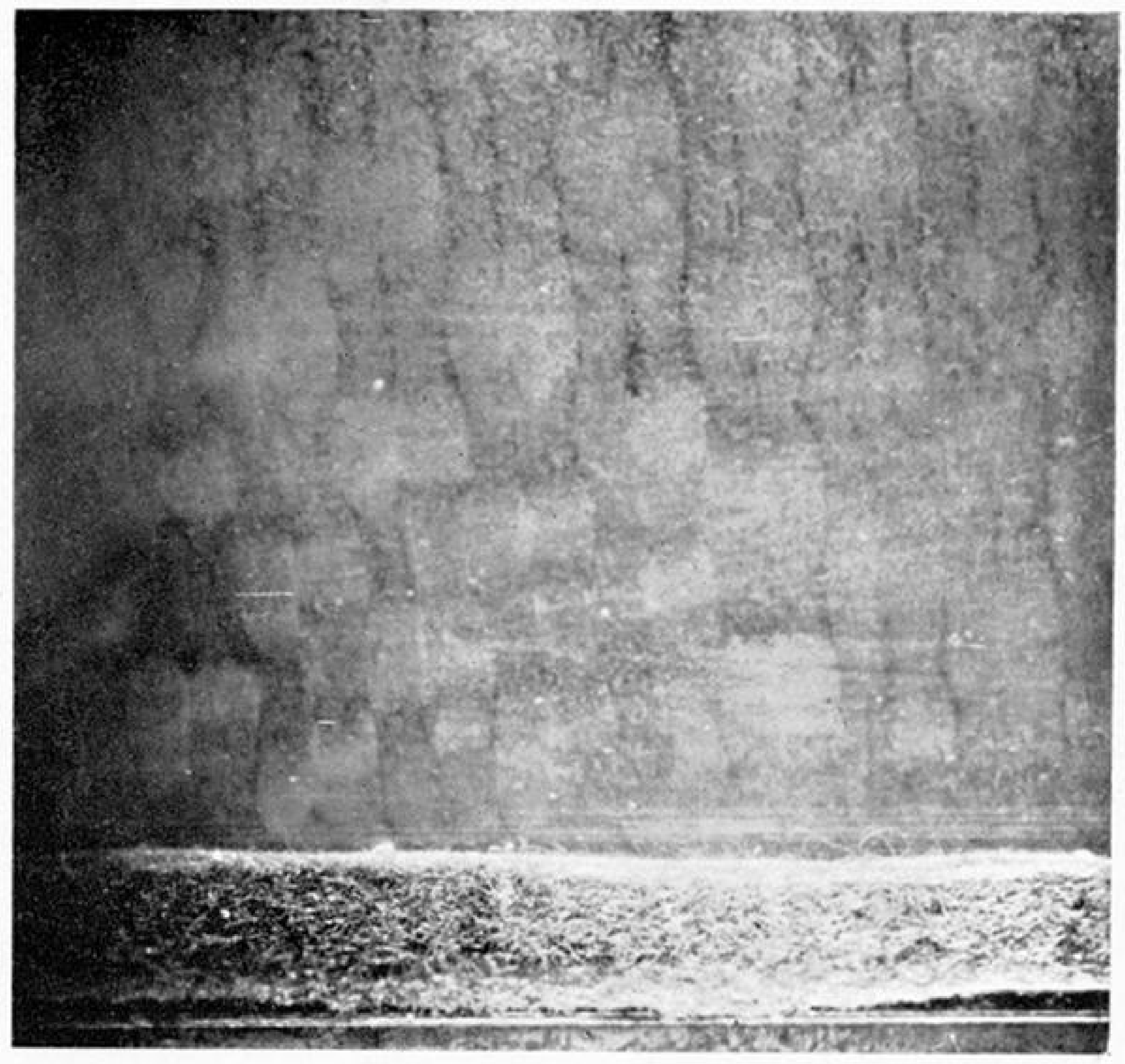

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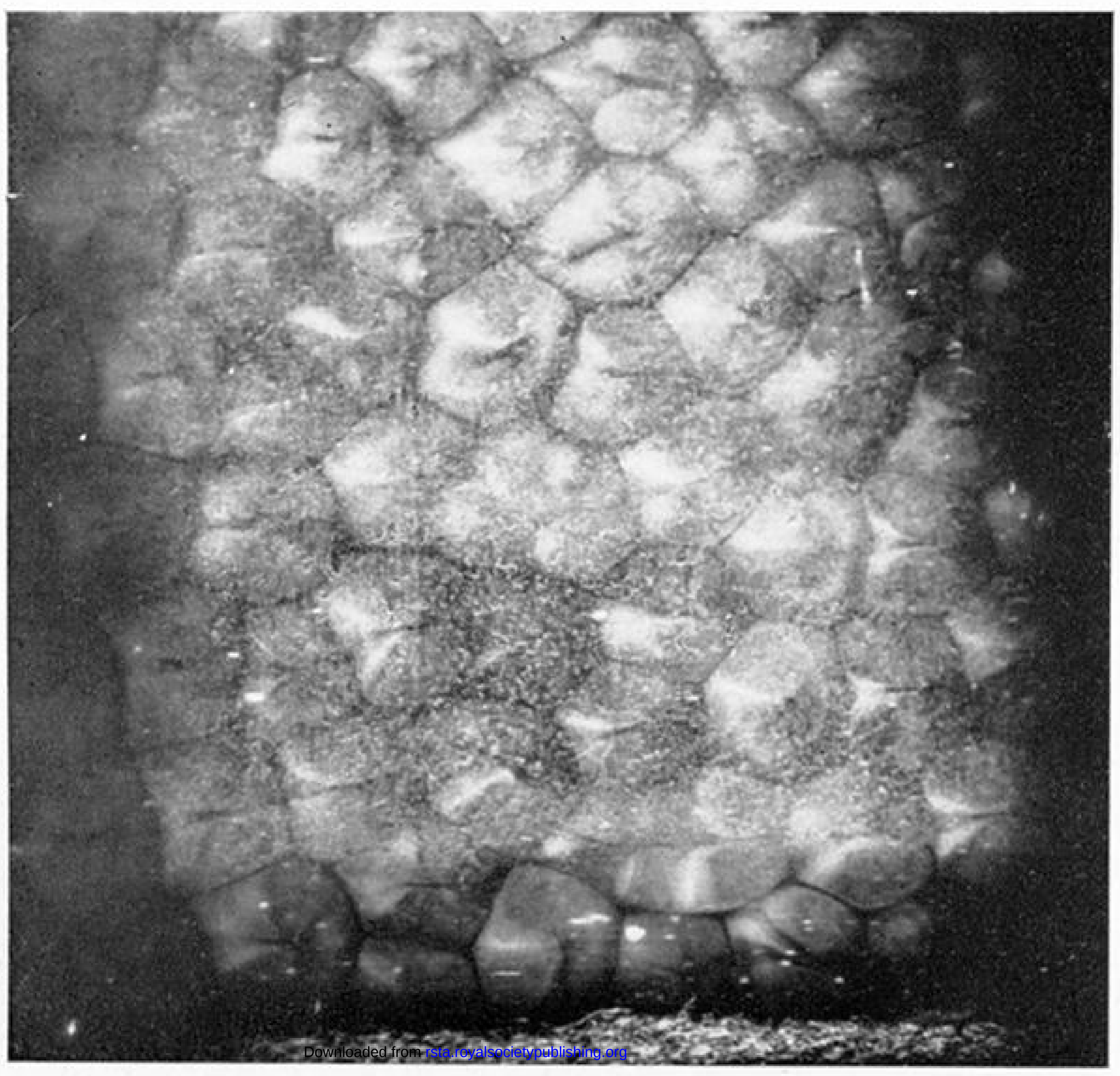

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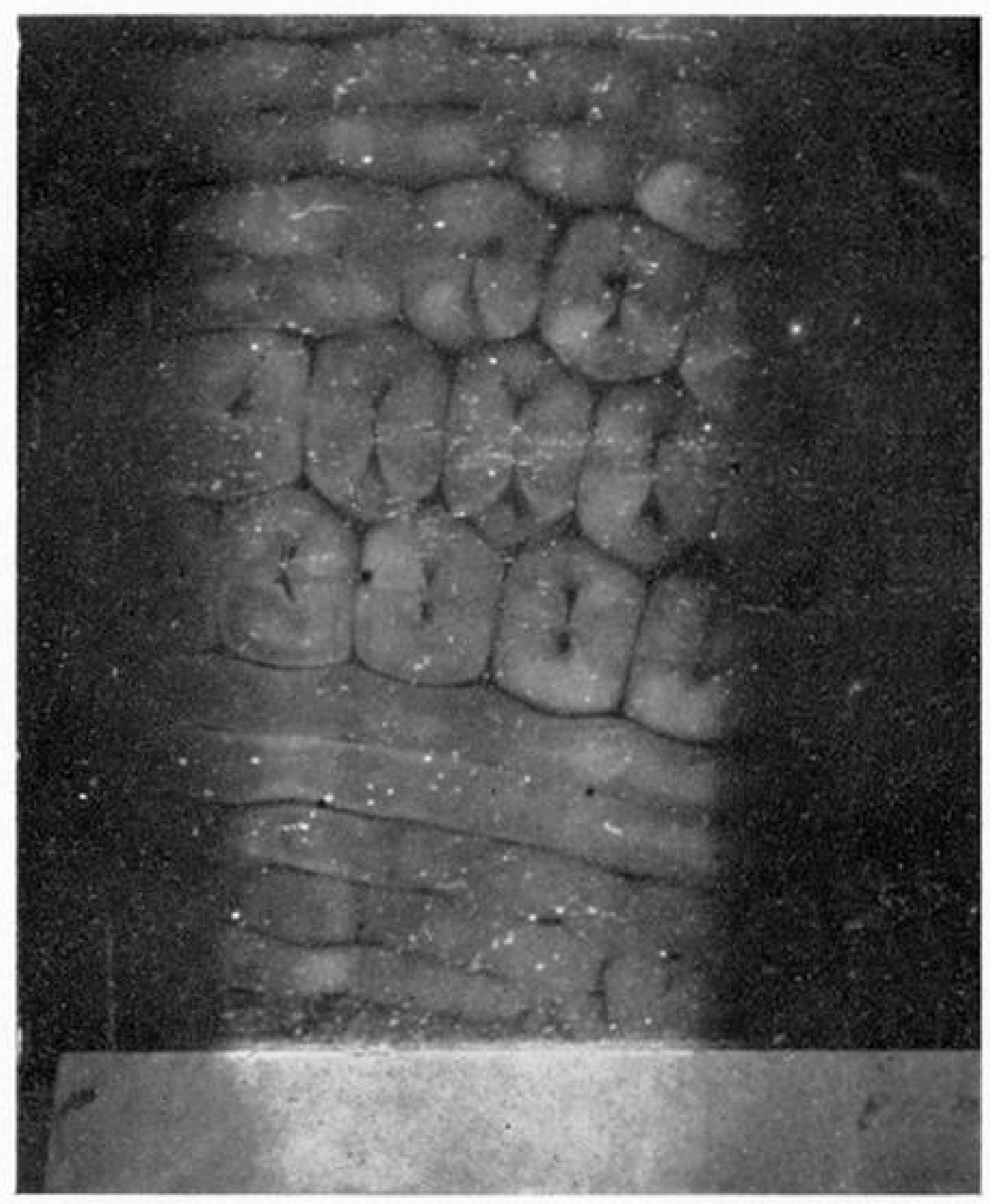

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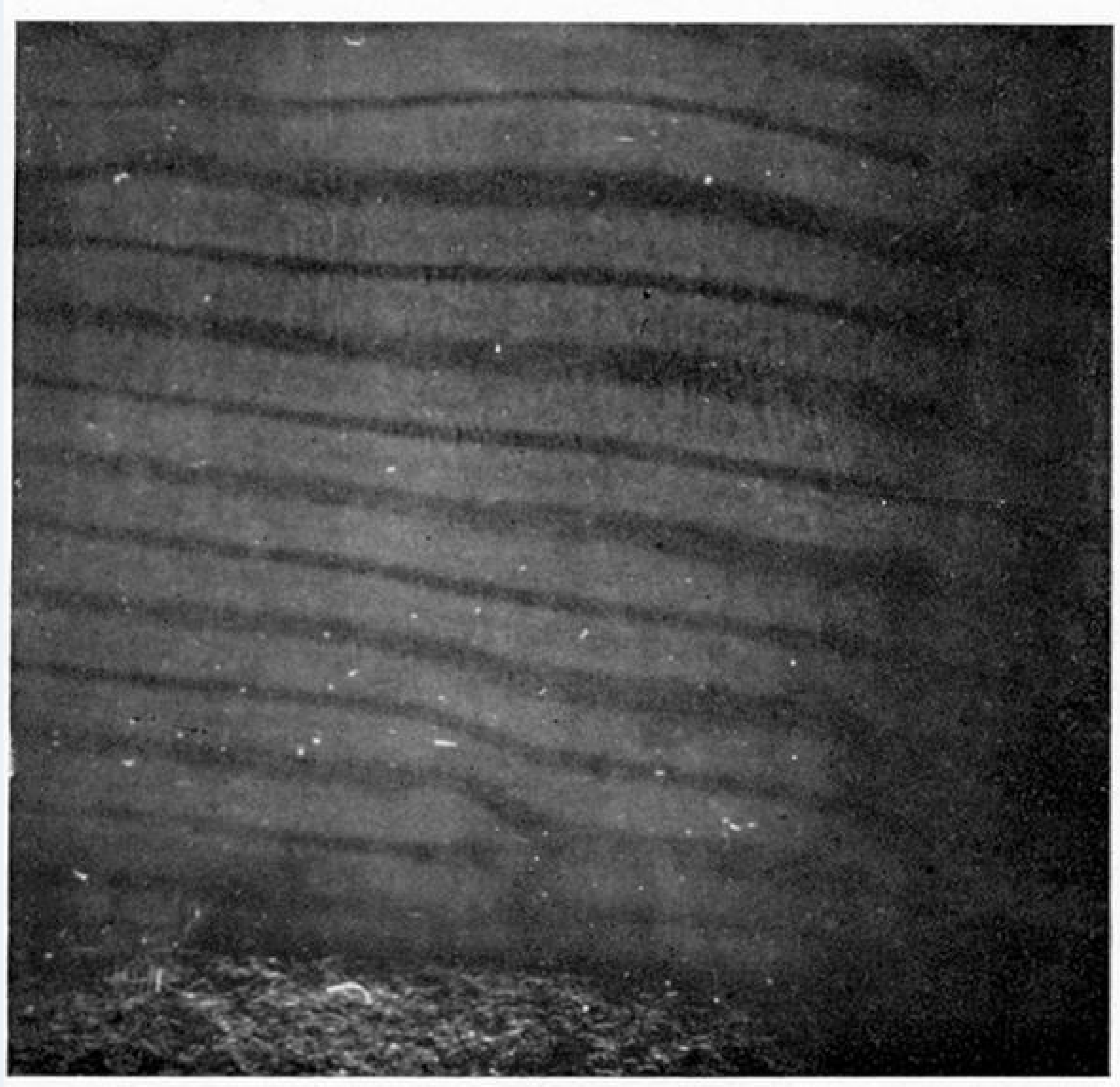

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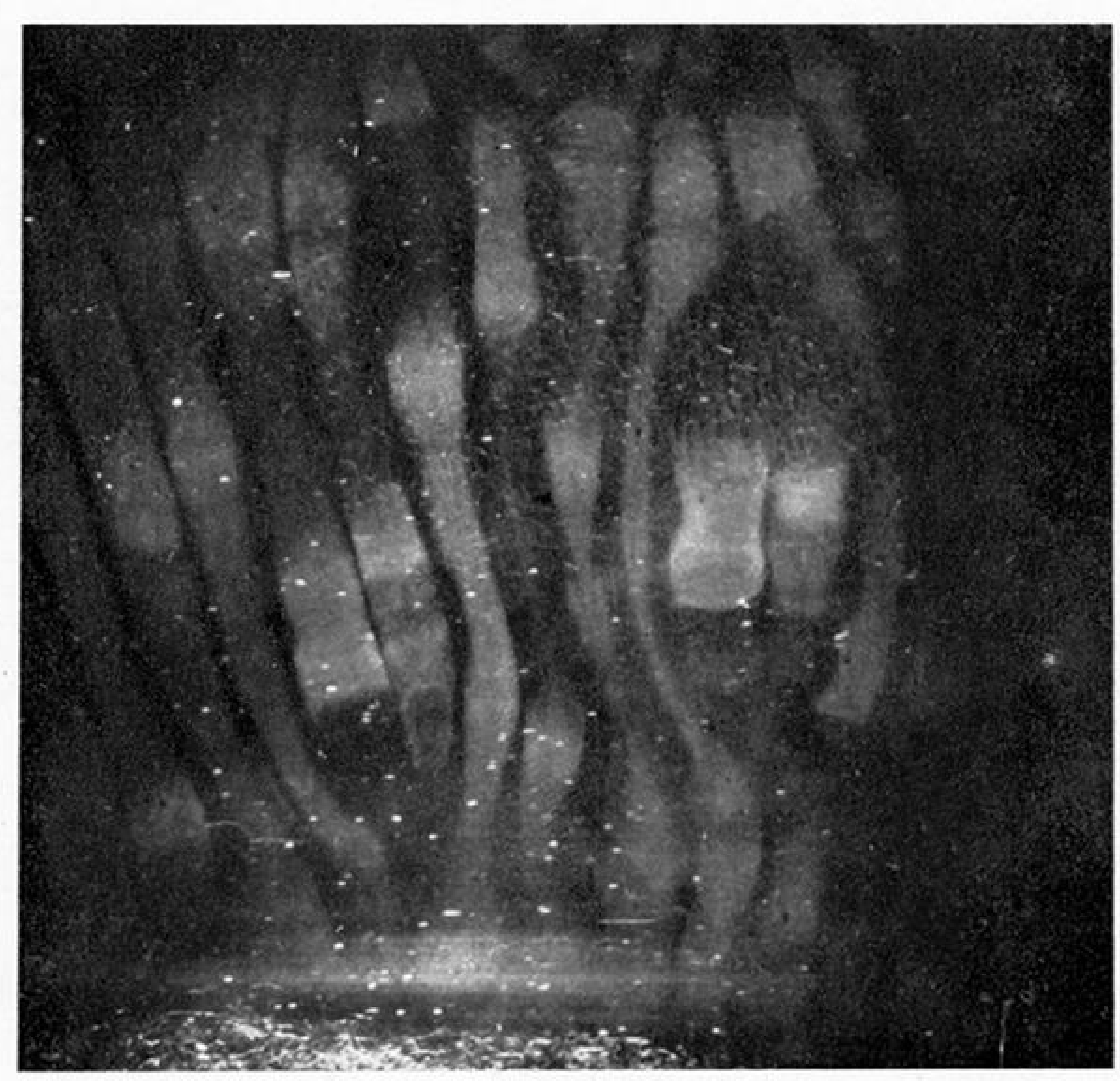

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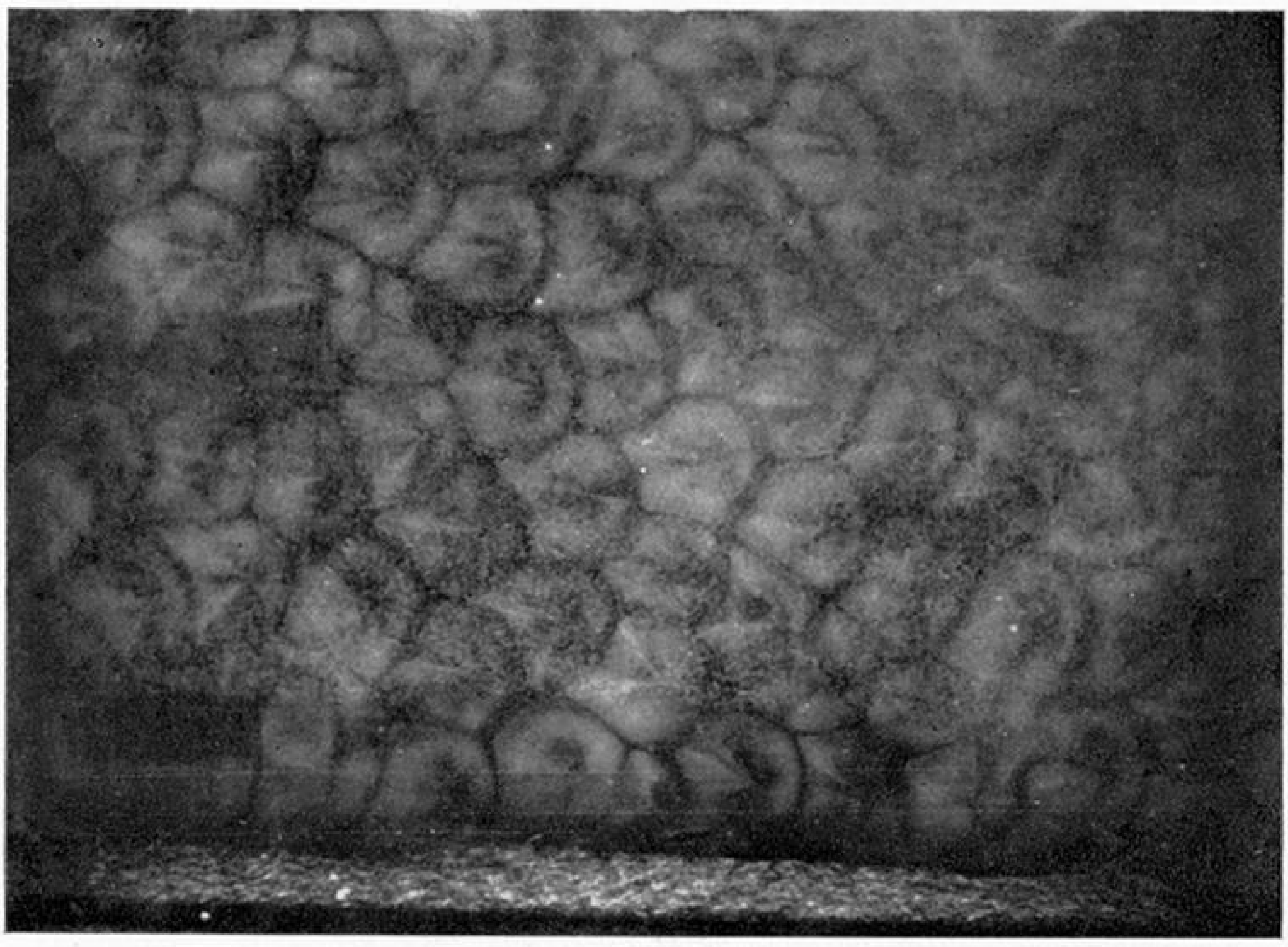

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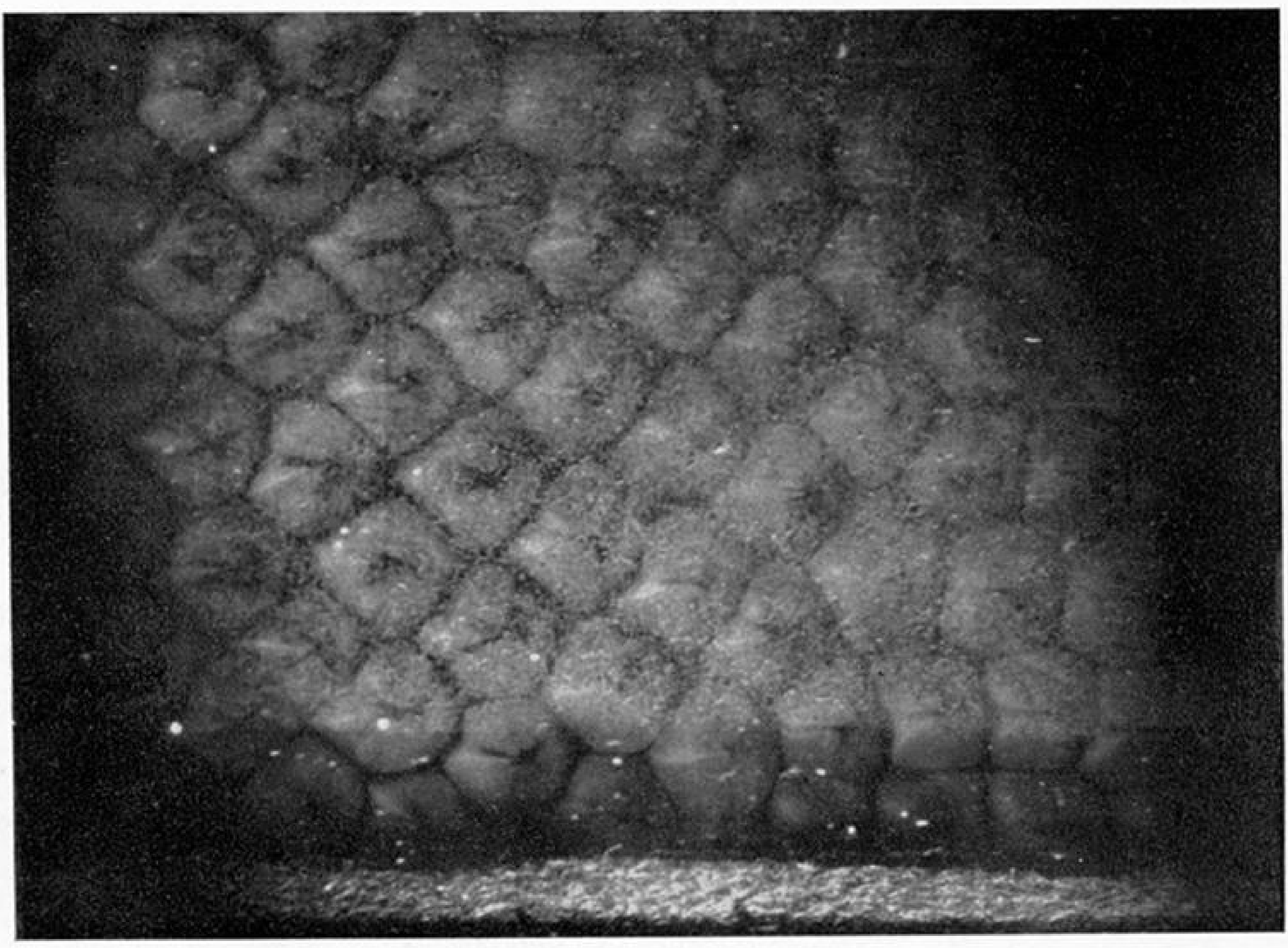

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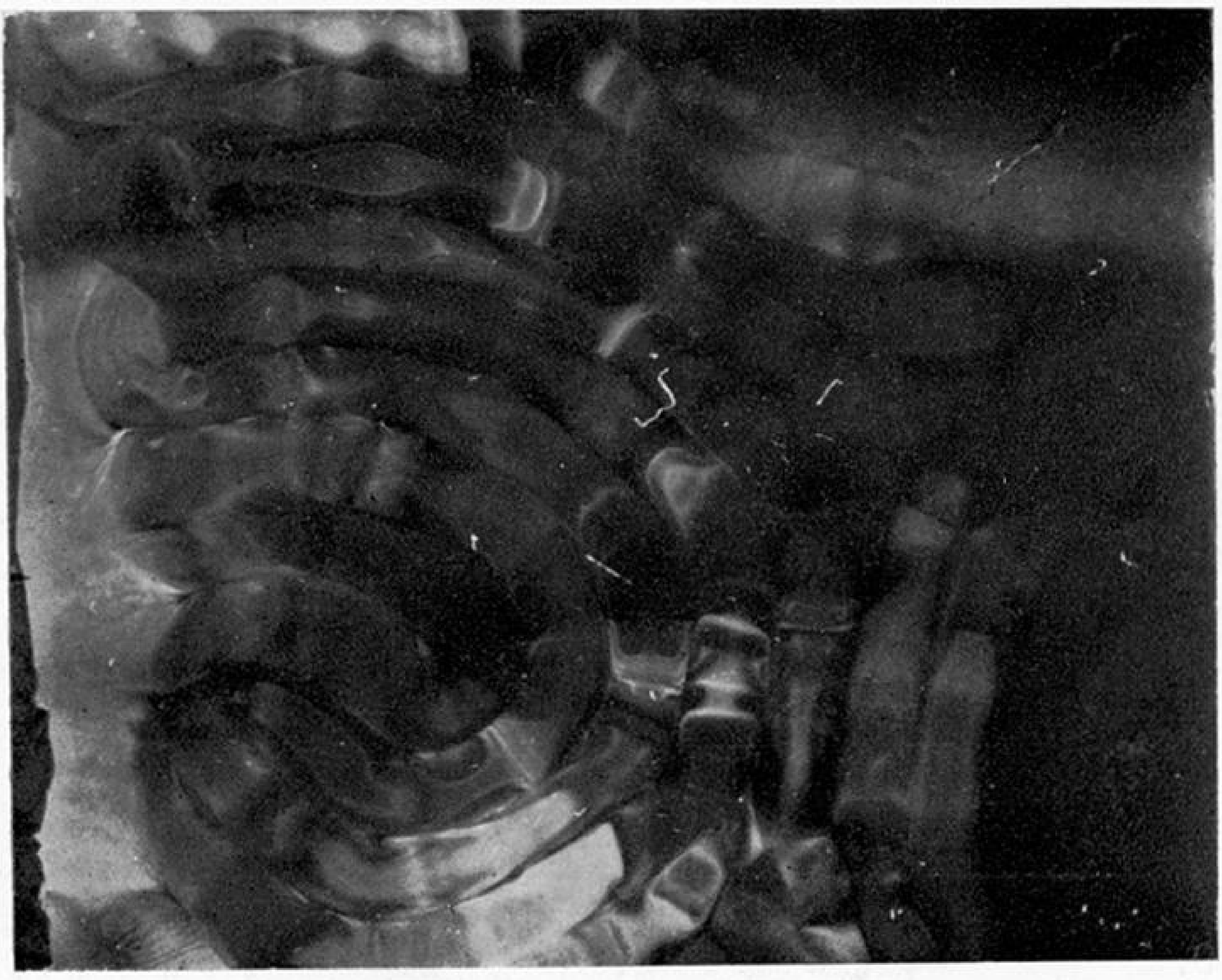

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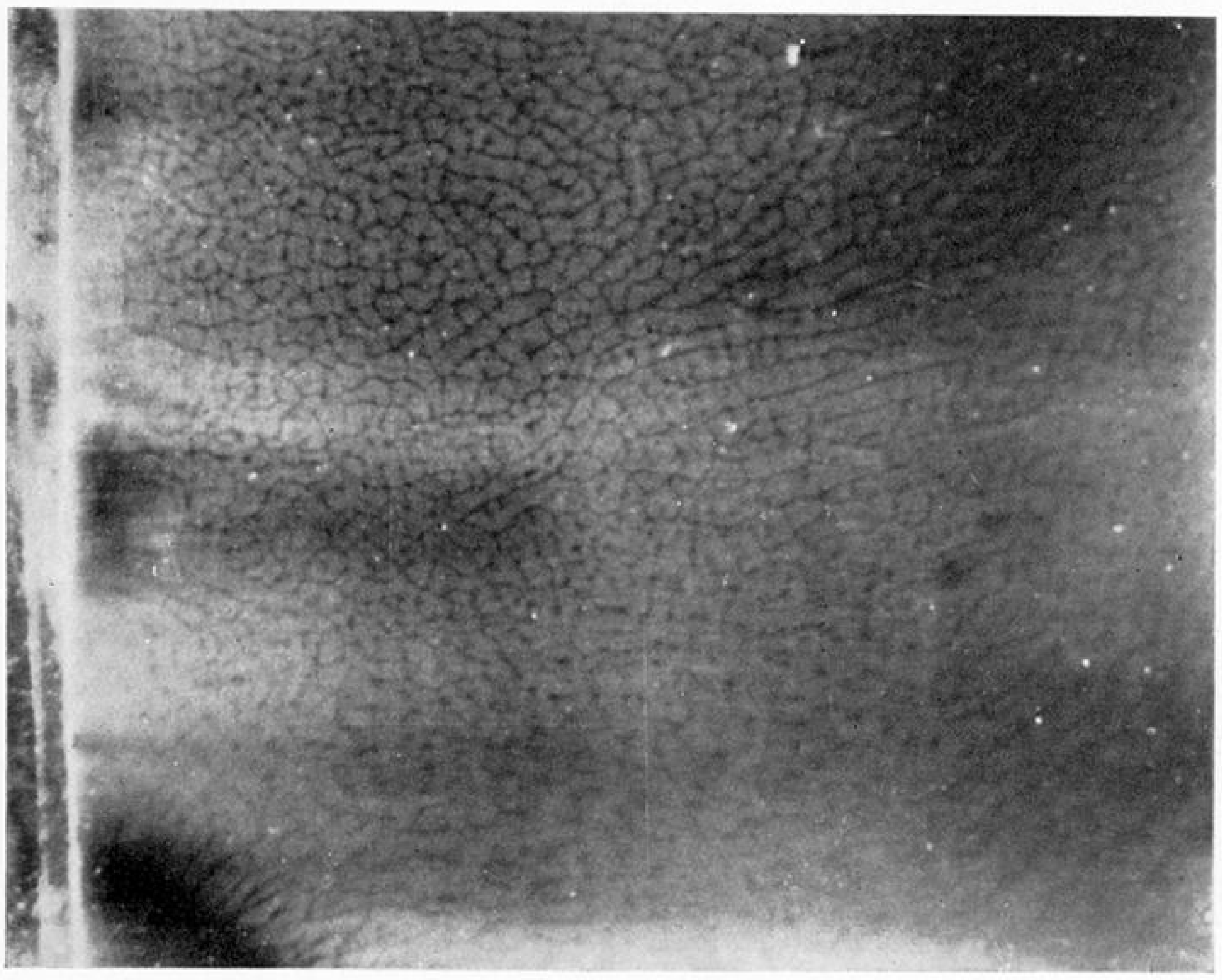
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
THE ROYAL SOCIETY A
PHILOSOPHICAL TRANSACTIONS OF



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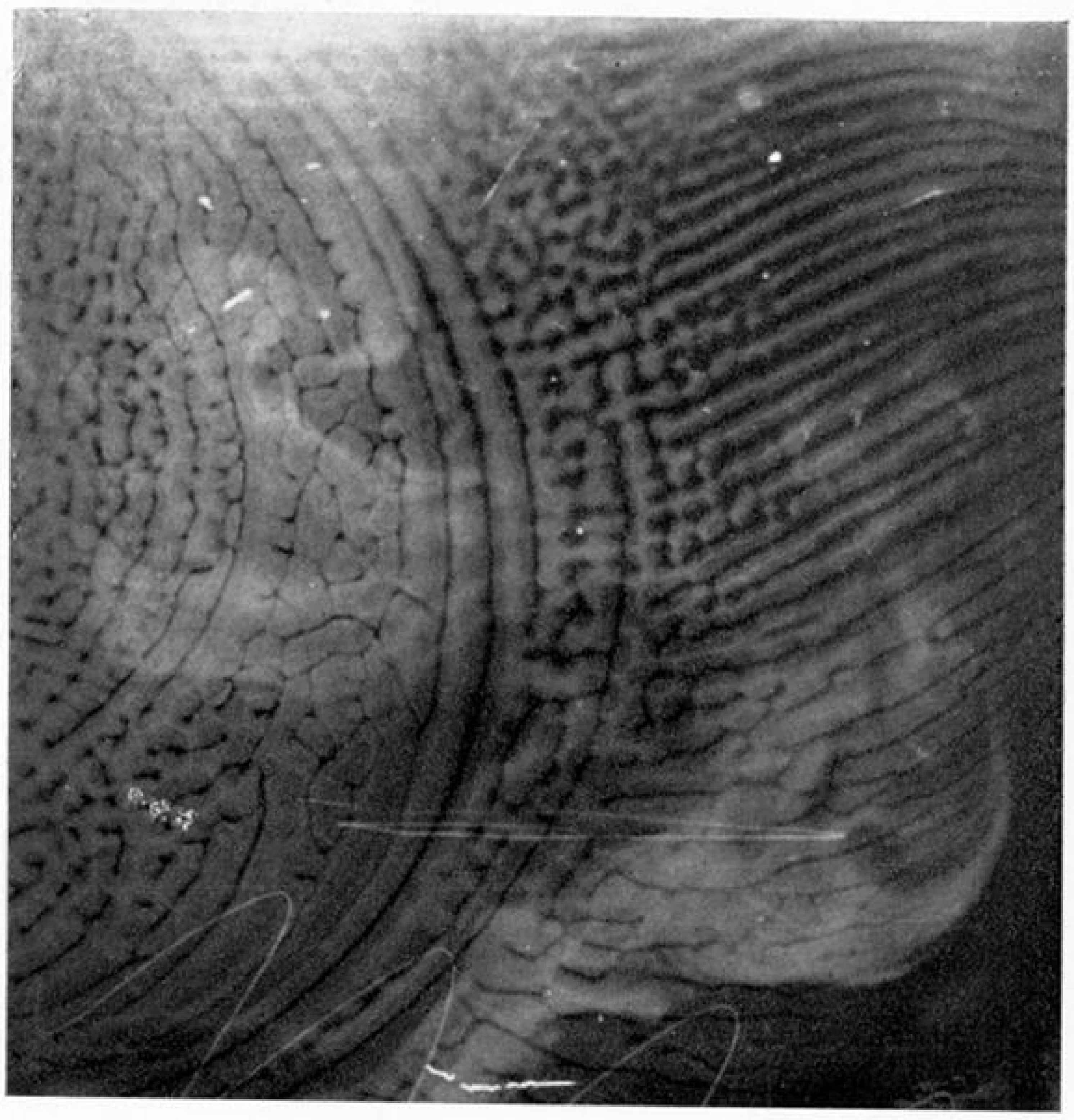

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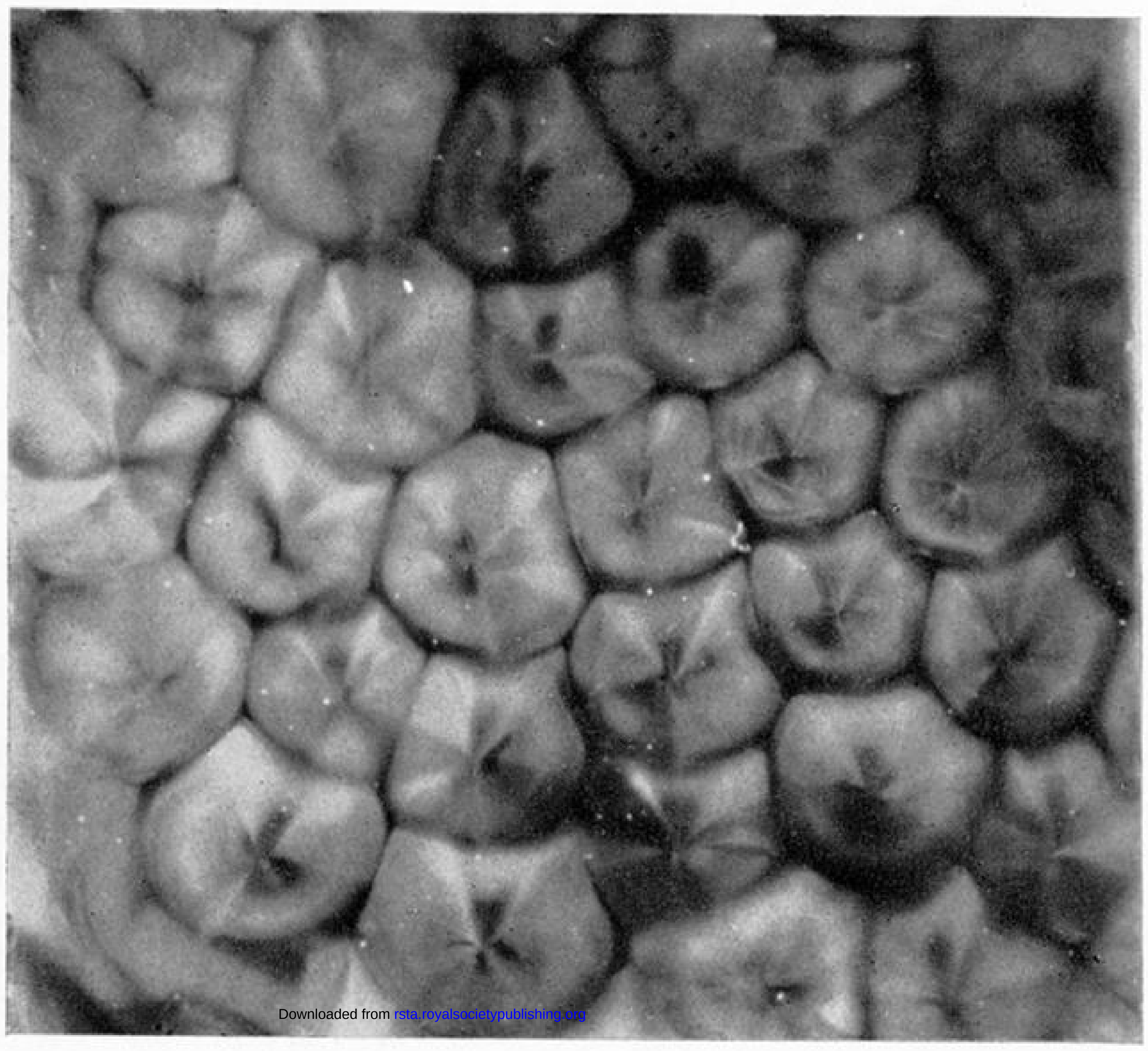

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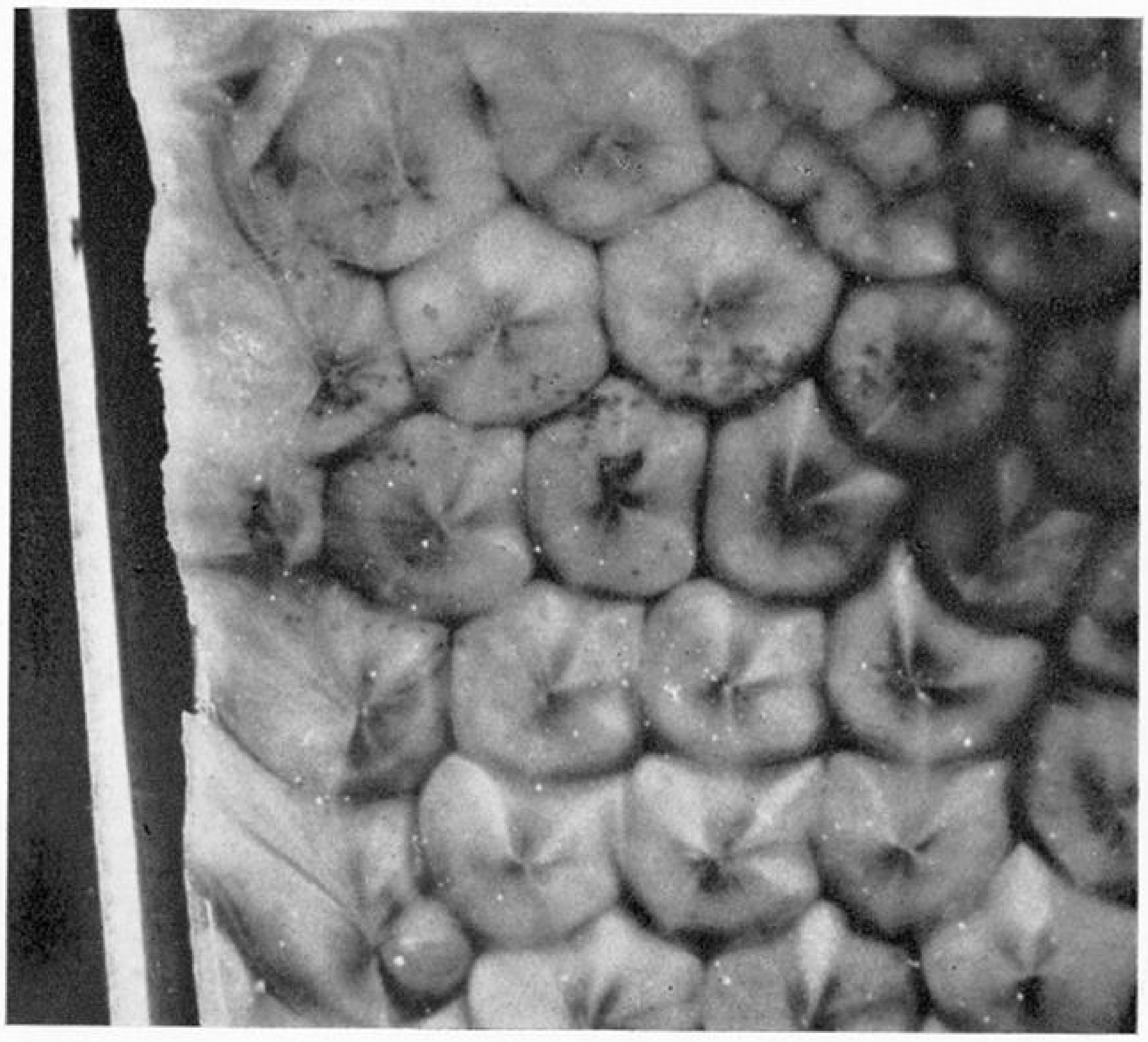

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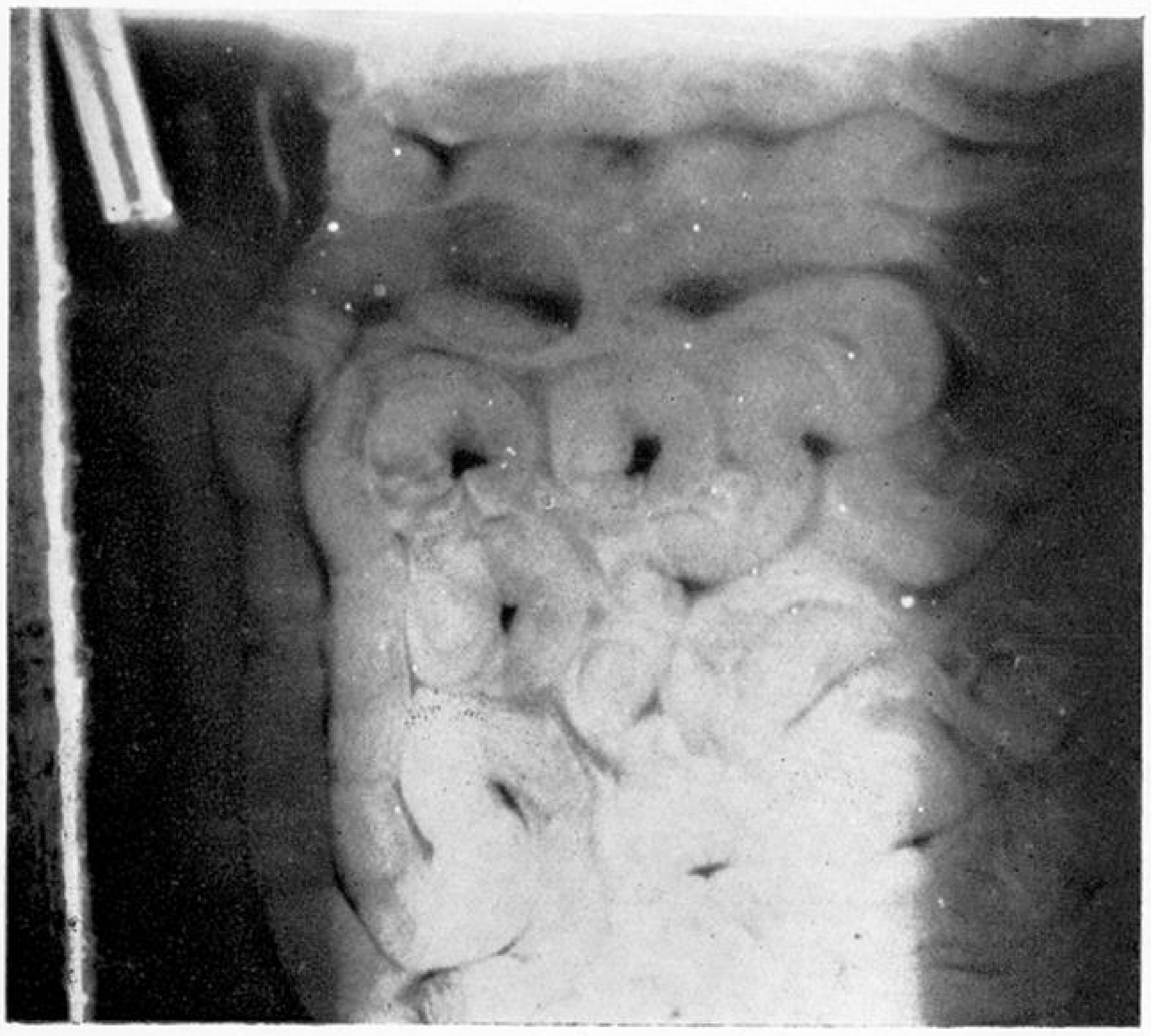

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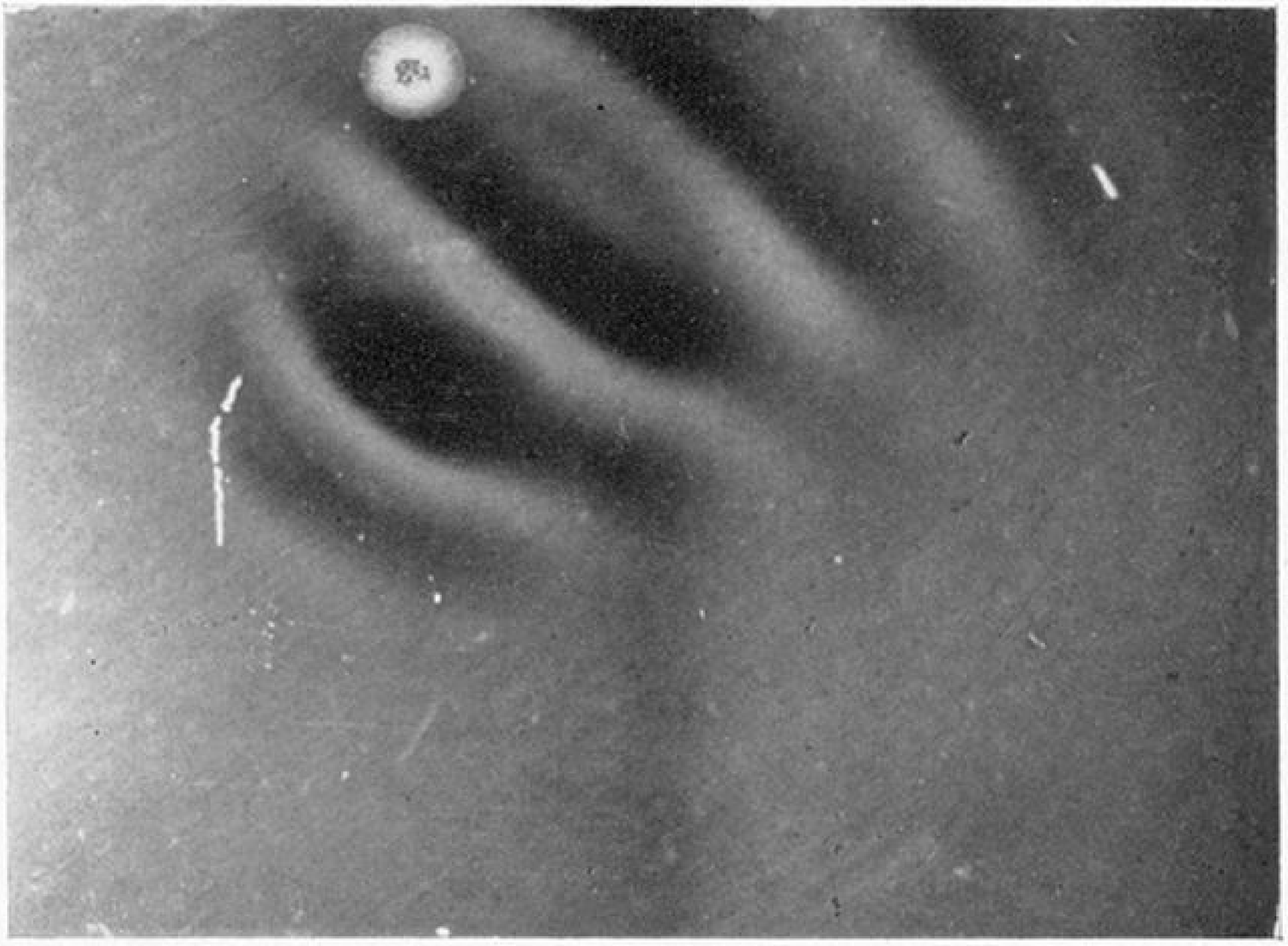

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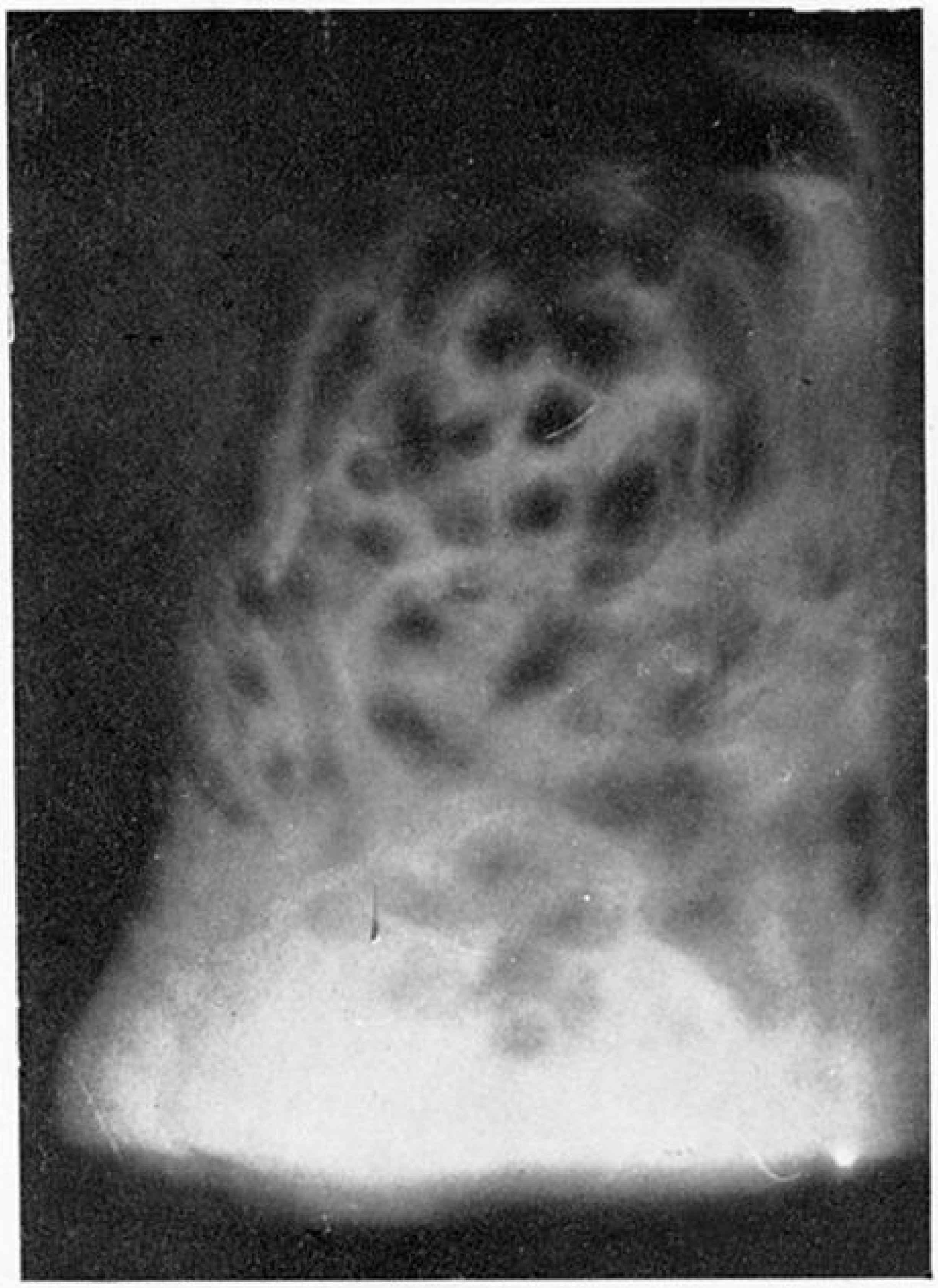

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